Journal of Green Engineering (JGE)

Volume-10, Issue-3, March 2020

# Statistical Hypothesis Test to the Fuzzy Samples from Biomedical Observations by Pivotal Spot of Trapezoidal Fuzzy Numbers

[1,*]K. S. Keerthika and [2]S. Parthiban

*[1,2]Division of Mathematics, Department of Science and Humanities, Vignan's Foundation for Science, Technology and Research, Vadlamudi, Guntur, Andhra Pradesh.India.*

*E-mail:[1]keerthikasundaram.29@gmail.com ,[2]sparthiban2008@gmail.com*

## Abstract

Test of hypothesis and decision making are generic and significant concluding part in every field. Arriving better result through statistical hypothesis test plays a vital role in almost all sectors such as industries, financial management, education, election commission, transports, natural resource development departments etc. But it is not always possible to have precise sample observations when the task is dealing with real time population. Practically, the observed samples may or may not be precise in nature. In this proposed work, some imprecise data of dengue and malarial victims have been taken and the concerned samples are observed in terms of fuzzy numbers; more generally in terms of trapezoidal fuzzy numbers (TrFNs). Further, the fuzzy numbers are defuzzified by a unique ranking method through pivotal spot of TrFNs. After defuzzification, relevant statistical method has been applied in the test of hypothesis to get better decision.

**Keywords:** Test of Hypothesis, Fuzzy numbers, Pivotal Spot, Rank, Samples.

*Journal of Green Engineering, Vol. 10_3, 1218–1231.*



Alpha Publishers

## 1 Introduction

Testing statistical hypotheses is an imperative tool for making decisions about earth problems. Fundamental information is generally considered correct, but it is usually much more sensible to take into account ambiguous properties that are inaccurate numbers. In this instance, the test statistic too gives an exact number. (Trapezoidal) This article presents a test method of statistical hypothesis based on alpha cut of fuzzy number.

Testing statistical hypotheses under ambiguous conditions focused on using the fuzzy hypotheses presented by Zadeh [1] by many developers. Abbasbandy et al. [2, 3] knew a technique to get the most accurate trapezoidal estimate of fuzzy numbers. Abhinav Bansal [4] examined specific mathematical assets of subjective trapezoidal fuzzy numbers. B. F. Arnold [5] experimented with new information on fuzzy hypotheses. Chachi et al. [6] described another way of dealing with the problem of sampling statistic hypotheses. Gajivaradhan et al. [7, 8, 9] conducted speculative experiments on various statistical concepts in an ambiguous scenario. P. Grzegorzewski [10, 11] has carried out various experiments in statistical theories with ambiguous numbers. Iuliana Carmen [12] tests true theories with the help of ambiguous etymological variables. Liou et al. [13] determination of the absolutely essential value of the ranking choice among ambiguous numbers. Salim Rezvani et al. [14] introduction of modelling and standardized mean coordinate representation (GMIR) for trapezoidal fuzzy numbers. Thorani et al. [15] with some modifications towards the positioning ability of a trapezoidal fuzzy number. Viertl [16] has studied some techniques to create final interpolations and statistical tests for ambiguous information. Wang et al. [17] introduction of the technique for key formulas for short fuzzy numbers. Wu [18, 19, 20] suggested several ways to develop certain fuzzy intervals for the fuzzy-fuzzy parameter. Excluded examples we propose another measurable fuzzy theory test for two-sided t-tests with information about trapezoidal fuzzy numbers. Another thought in this work, is that we have an idea of an investigation, and what will be the consequences if we reach the level of this relaxed information that is essential? As a result, we used positions in theoretical experiments that were derived from the proposed critical positions of trapezoidal fuzzy numbers. Confidence levels and h-level sentences are not used in the selection guidelines for the proposed test procedure.

# 2 Preliminaries

## *2.1 Membership Function*

A fuzzy set $\widetilde{A}$ of a universal set X is characterized by its membership function $\mu_{\tilde{A}} : X \to [0, 1]$ and we write $\widetilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$.

## *2.2 Convex Fuzzy Set*

A fuzzy set $\widetilde{A}$ is said to be a convex fuzzy set if $\mu_{\tilde{A}}(\alpha x_1 + (1 - \alpha) x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ where $x_1, x_2 \in X$ and $\alpha \in [0, 1]$.

## *2.3 Fuzzy Number*

A fuzzy set $\widetilde{A}$ defined on the universal set of real number R, whose membership function $\mu$ is called the fuzzy number if it has the following characteristics:

i. $\widetilde{A}$ is convex,

ii. $\widetilde{A}$ is normal,

iii. $\mu_{\tilde{A}}$ is piecewise continuous.

## *2.4 Normalized Trapezoidal Fuzzy Number*

A fuzzy number $\widetilde{A} = (m, n, p, q; 1)$ is a Normalized Trapezoidal Fuzzy Number if its membership function is specified by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; \; x < m \\ \dfrac{x - m}{n - m} & ; \; m < x \leq n \\ 1 & ; \; n < x < p \\ \dfrac{q - x}{q - p} & ; \; p \leq x < q \\ 0 & ; \; x > q \end{cases} \quad \text{where } m \leq n \leq p \leq q$$

## 3 Hypothesis Test for Interval Data

Let $\{[a_i, b_i]$ , i =1, 2……, n} be a random sample (x) with size $m$ and $\{[c_j, d_j]$, j=1, 2,…….., n} be a random sample (y) with size $n$. And consider $[\eta_1, \mu_1]$ be mean of $x$ from a normal population, also consider $[\eta_2, \mu_2]$ be present mean of $y$ as of one more normal population.

Moreover, the null hypothesis $H_o$ so that the mean values of the population of the given samples are the same that is,
$H_0 : [\eta_1, \mu_1] = [\eta_2, \mu_2] \Rightarrow \eta_1 = \eta_2$ and $\mu_1 = \mu_2$

And the alternative hypothesis is given by

$$(i) H_A : [\eta_1, \mu_1] < [\eta_2, \mu_2] \Rightarrow \eta_1 < \eta_2 \text{ and } \mu_1 < \mu_2$$

$$(ii) H_A : [\eta_1, \mu_1] > [\eta_2, \mu_2] \Rightarrow \eta_1 > \eta_2 \text{ and } \mu_1 > \mu_2$$

$$(iii) H_A : [\eta_1, \mu_1] \neq [\eta_2, \mu_2] \Rightarrow \eta_1 \neq \eta_2 \text{ and } \mu_1 \neq \mu_2$$

Suppose that $\tilde{x}$ and $\tilde{y}$ be the sample means, $s_{\tilde{x}}$ and $s_{\tilde{y}}$ are the sample standard deviations of $\tilde{x}$ and $\tilde{y}$ correspondingly.

**Case (i)**

Let presumed the population standard deviation is to be same, then the null hypothesis $H_0 : [\eta_1, \mu_1] = [\eta_2, \mu_2]$ test will be

$$\tilde{t} = \frac{\tilde{\tilde{x}} - \tilde{\tilde{y}}}{s\sqrt{\frac{1}{m} + \frac{1}{n}}} \tag{1}$$

where

$$\tilde{s} = \sqrt{\frac{(m-1)s^2_{\tilde{x}} + (n-1)s^2_{\tilde{y}}}{m+n-2}} \tag{2}$$

**Case (ii)**

Let presumed the population standard deviation is to be in-equal, then the null hypothesis $H_0 : [\eta_1, \mu_1] \neq [\eta_2, \mu_2]$ test will be

$$\tilde{t} = \frac{\overline{\tilde{\tilde{x}}} - \overline{\tilde{\tilde{y}}}}{\sqrt{\frac{s^2_{\tilde{x}}}{m} + \frac{s^2_{\tilde{y}}}{n}}} \tag{3}$$

where

$$s^2_{\tilde{x}} = \left(\frac{1}{m-1}\right)\left(\sum_{i=1}^{m}\left(\widetilde{x_i} - \tilde{\tilde{x}}\right)^2\right) \text{ and } s^2_{\tilde{y}} = \left(\frac{1}{n-1}\right)\left(\sum_{j=1}^{n}\left(\widetilde{y_j} - \tilde{\tilde{y}}\right)^2\right) \tag{4}$$

Here, the parameters are denoted with tilde $(\sim)$ symbol as they are defuzzified form connected with NTrFN.

## 4 The Pivotal Spot for NTrFN

Consider a normalized trapezoidal fuzzy number (NTrFN) $\widetilde{T} = (t_1, t_2, t_3, t_4; 1)$. In the proposed method, this NTrFN has been divided into four plane figures namely triangle ABC, triangle ABD, triangle ACD and triangle BCD. The centroid of these four triangles has been found, these centroids are $m_1, m_{2,} m_3$ and $m_4$. Consequently, the four centroids form a four-sided polygon with the vertices $m_1, m_{2,} m_3$ and $m_4$. In this study, these locuses are taken as the better perspectives to locate the most pivotal spot of NTrFN $\widetilde{T}$. The proposed pivotal spot is the point of intersection of the diagonals $d_1$ and $d_2$ joining the vertices $\overline{m_1 m_3}$ and $\overline{m_2 m_4}$ respectively. That is $d_1 = \overline{m_1 m_3}$ and $d_2 = \overline{m_2 m_4}$. As this pivotal spot is taken out from the centroids of four triangles derived from the NTrFN, this would be a much more balancing point for a normalized trapezoidal fuzzy number. The Cartesian form of the pivotal spot of NTrFN has been finally formed.

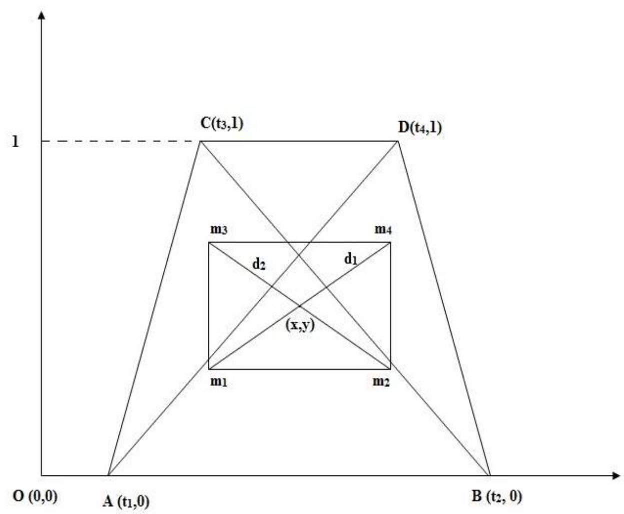


**Figure 1** Pivotal Spot of NTrFN

From figure 1, it is seen that the centroids of the four plane figures are:

$$m_1=\left(\frac{t_1+t_2+t_3}{3},\frac{1}{3}\right),\ m_2=\left(\frac{t_1+t_2+t_4}{3},\frac{1}{3}\right),\ m_3=\left(\frac{t_1+t_3+t_4}{3},\frac{2}{3}\right)\text{ and } m_4=\left(\frac{t_2+t_3+t_4}{3},\frac{2}{3}\right)$$

The pivotal spot of the polygon is derived as follows,

Consider a normalized trapezoidal fuzzy number as

$$\widetilde{T}=\left(t_1,t_2,t_3,t_4;1\right) \tag{5}$$

And the centroid of the four triangles has taken as follows

$$\Delta\, ABC, m_1=\left(\frac{t_1+t_2+t_3}{3},\frac{1}{3}\right)=m_1\left(s_1,s_2\right) \tag{6}$$

$$\Delta\, ABD, m_2=\left(\frac{t_1+t_2+t_4}{3},\frac{1}{3}\right)=m_2\left(s_3,s_4\right) \tag{7}$$

$$\Delta\, ACD, m_3=\left(\frac{t_1+t_3+t_4}{3},\frac{2}{3}\right)=m_3\left(s_5,s_6\right) \tag{8}$$

$$\Delta\, BCD, m_4 = \left( \frac{t_2 + t_3 + t_4}{3}, \frac{2}{3} \right) = m_4\left(s_7, s_8\right) \tag{9}$$

The diagonal equations of $d_1$ and $d_2$ formulated by point-slope formulas as follows

$$y - y_1 = m(x - x_1) \text{ where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore\ y - y_1 = \left( \frac{y_2 - y_1}{x_2 - x_1} \right)(x - x_1) \tag{10}$$

Now by substituting (6) and (9) in (10), the diagonal equation $d_1$ will be

$$y - s_2 = \left( \frac{s_8 - s_2}{s_7 - s_1} \right)(x - s_1) \tag{11}$$

Similarly, by substituting (7) and (8) in (10), the diagonal equation $d_2$ will be

$$y - s_4 = \left( \frac{s_6 - s_4}{s_5 - s_3} \right)(x - s_3) \tag{12}$$

Substituting equations (6), (7), (8) and (9) in equations (11) and (12), we get two equations. That is given below

$$y - \frac{1}{3} = \left[ \frac{\frac{2}{3} - \frac{1}{3}}{\left( \frac{t_2 + t_3 + t_4}{3} \right) - \left( \frac{t_1 + t_2 + t_3}{3} \right)} \right] \left( x - \frac{t_1 + t_2 + t_3}{3} \right) \tag{13}$$

$$y - \frac{1}{3} = \left[ \frac{\frac{2}{3} - \frac{1}{3}}{\left( \frac{t_1 + t_3 + t_4}{3} \right) - \left( \frac{t_1 + t_2 + t_4}{3} \right)} \right] \left( x - \frac{t_1 + t_2 + t_4}{3} \right) \tag{14}$$

Solving equations (13) and (14) for x and y successively, it is seen that the pivotal spot $P(\tilde{x},\tilde{y})$ of NTrFN $\tilde{T}$ will be of the form

$$\left(\tilde{x},\tilde{y}\right)=\left[\frac{t_1^2-t_2^2+t_3^2-t_4^2+t_1t_3-t_2t_4}{3(t_1-t_2+t_3-t_4)},\frac{t_2^2-2t_3^2+t_3(3t_2+2t_4-t_1)+t_2(2t_4-t_1)}{3(t_3-t_2)(t_1-t_2+t_3-t_4)}\right] \quad (15)$$

The rank of the NTrFN is defined by its geometric mean of the pivotal-points is

$$\tilde{\tilde{x}}_D \text{ and } \tilde{\tilde{y}}_M,\ R(\tilde{T})=\sqrt{\tilde{x}^2+\tilde{y}^2}\ .$$

Let $T_{\tilde{A}}$ and $T_{\tilde{B}}$ be any two TrFNs then it can be proved the following results,

$$(i) R(T_{\tilde{A}})=R(T_{\tilde{B}})\Rightarrow T_{\tilde{A}}=T_{\tilde{B}}$$
$$(ii) R(T_{\tilde{A}})<R(T_{\tilde{B}})\Rightarrow T_{\tilde{A}}<T_{\tilde{B}}$$
$$(iii) R(T_{\tilde{A}})>R(T_{\tilde{B}})\Rightarrow T_{\tilde{A}}>T_{\tilde{B}}$$
$$(iv) R(T_{-\tilde{A}})>R(T_{-\tilde{B}})\Rightarrow T_{-\tilde{A}}>T_{-\tilde{B}}$$

## 5 Numerical Example

The following data has been observed by a health inspection committee in a street of a rural area in Tamil Nadu on disease among people in a particular area. Due to some unavoidable situation, complete enumeration about all affected people could not be possible. Therefore, the intensity of the effect of the disease based on four stages is recorded as normalized trapezoidal fuzzy numbers that has been given below. For this problem, the test of hypothesis has been conducted to test the homogeneity of the severity of the infection the two diseases. Here the given data are in the form of NTrFN in table 1. These NTrFN are transformed to interval data by using pivotal spot defuzzifying formula (15) given in Section-4.

**Table 1** Fuzzy Samples in terms of TrFNs

| **Victims of Dengue infection** | **Victims of Malarial infection** |
|---|---|
| $\tilde{T}_1$ = (0.2, 0.5, 0.7, 0.9) | $\tilde{T}_{11}$ =(0.5, 0.7, 0.8, 0.9) |
| $\tilde{T}_2$ = (0.1, 0.4, 0.8, 0.9) | $\tilde{T}_{12}$ =(0.1, 0.4, 0.5, 0.6) |
| $\tilde{T}_3$ =(0.2, 0.3, 0.6, 0.8) | $\tilde{T}_{13}$ =(0.3, 0.5, 0.6, 0.8) |
| $\tilde{T}_4$ =(0.3, 0.5, 0.7, 0.8) | $\tilde{T}_{14}$ =(0.2, 0.4, 0.6, 0.7) |
| $\tilde{T}_5$ =(0.3, 0.4, 0.7, 0.9) | $\tilde{T}_{15}$ =(0.1, 0.2, 0.5, 0.8) |
| $\tilde{T}_6$ =(0.3, 0.4, 0.6, 0.8) | $\tilde{T}_{16}$ =(0.3, 0.5, 0.6, 0.9) |
| $\tilde{T}_7$ =(0.2, 0.6, 0.7, 0.8) | $\tilde{T}_{17}$ =(0.1, 0.2, 0.7, 0.9) |
| $\tilde{T}_8$ =(0.3, 0.5, 0.6, 0.7) | $\tilde{T}_{18}$ =(0.2, 0.3, 0.6, 0.8) |
| $\tilde{T}_9$ =(0.3, 0.5, 0.6, 0.8) | $\tilde{T}_{19}$ =(0.2, 0.3, 0.6, 0.7) |
| $\tilde{T}_{10}$ =(0.3, 0.5, 0.7, 0.9) | $\tilde{T}_{20}$ =(0.3, 0.5, 0.7, 0.8) |

**The Null Hypothesis**

$\widetilde{H}_0$ : The intensity of the infection of two diseases is same.

**The Alternative Hypothesis**

$\widetilde{H}_A$ : There is a significant difference between the severity of the infection of the two disease

The test statistics and other parameters are denoted by

$\tilde{T}_1 = (0.2, 0.5, 0.7, 0.9)$

$$\tilde{x} = \frac{t_1^2 - t_2^2 + t_3^2 - t_4^2 + t_1 t_3 - t_2 t_4}{3(t_1 - t_2 + t_3 - t_4)}$$

$$\tilde{x} = \frac{0.2^2 - 0.5^2 + 0.7^2 - 0.9^2 + (0.2)(0.7) - (0.5)(0.9)}{3(0.2 - 0.5 + 0.7 - 0.9)}$$

$\tilde{x} = 0.560000$

$$\tilde{y} = \frac{t_2^2 + 2t_3^2 - t_3(3t_2 + 2t_4 - t_1) + t_2(2t_4 - t_1)}{3(t_3 - t_2)(t_1 - t_2 + t_3 - t_4)}$$

$$\tilde{y} = \frac{0.2^2 + 2(0.7^2) - 0.7(3\times 0.5 + 2\times 0.9 - 0.2) + 0.5(2\times 0.9 - 0.2)}{3(0.7 - 0.5)(0.2 - 0.5 + 0.7 - 0.9)}$$

$\tilde{y} = 0.466667$

The interval form of $\tilde{T}_1$ is $\tilde{I}_1 = [\tilde{x}, \tilde{y}]$ (i.e.) $\tilde{I}_1 = [0.560000, 0.466667]$ Similarly, the other observations can be transformed to interval form. The transformed interval data is tabulated in table 2, For $\tilde{T}_i; i = 1, 2, ...., 20$.

**Table 2** Interval Data of Fuzzy Samples

| Dengue(x) | | Malaria(y) | |
|---|---|---|---|
| $\tilde{T}_1$ | [0.560000, 0.466667] | $\tilde{T}_{11}$ | [0.711111, 0.444444] |
| $\tilde{T}_2$ | [0.500000, 0.416667] | $\tilde{T}_{12}$ | [0.375000, 0.416667] |
| $\tilde{T}_3$ | [0.500000, 0.555556] | $\tilde{T}_{13}$ | [0.550000, 0.500000] |
| $\tilde{T}_4$ | [0.555556, 0.444444] | $\tilde{T}_{14}$ | [0.455556, 0.444444] |
| $\tilde{T}_5$ | [0.600000, 0.555556] | $\tilde{T}_{15}$ | [0.441667, 0.583333] |
| $\tilde{T}_6$ | [0.544444, 0.555556] | $\tilde{T}_{16}$ | [0.586667, 0.533333] |
| $\tilde{T}_7$ | [0.540000, 0.400000] | $\tilde{T}_{17}$ | [0.511111, 0.555556] |
| $\tilde{T}_8$ | [0.511111, 0.444444] | $\tilde{T}_{18}$ | [0.500000, 0.555556] |
| $\tilde{T}_9$ | [0.550000, 0.500000] | $\tilde{T}_{19}$ | [0.450000, 0.500000] |
| $\tilde{T}_{10}$ | [0.600000, 0.500000] | $\tilde{T}_{20}$ | [0.555556, 0.444444] |

Based on the interval data calculated above, the test of hypotheses will be evaluated as per the test statistics given in Section-2.

**Rank of Normalized Trapezoidal Fuzzy Numbers**

The rank of the NTrFN is defined by its geometric mean of the ***pivotal-points:*** $\tilde{\tilde{x}}_D$ and $\tilde{\tilde{y}}_M$, $R(\tilde{T}) = \sqrt{\tilde{x}^2 + \tilde{y}^2}$

Now, the ranks of NTrFN will be evaluated for the above numerical example by using the following formula:

$$R(\tilde{T}) = \sqrt{\tilde{x}^2 + \tilde{y}^2}$$

$$R(\tilde{T}_1) = \sqrt{0.560000 + 0.466667}$$

$$R(\tilde{T}_1) = 0.728957$$

Similarly, $R(\tilde{T}_i); i = 1,2,3,.....,20$ have calculated and tabulated in table 3:

**Table 3** Rank of Fuzzy Samples

| $R(\tilde{T}_i); i = 1,2,3,.....,20$ | |
|---|---|
| **Dengue(x)** | **Malaria(y)** |
| 0.728957 | 0.838576 |
| 0.650854 | 0.560568 |
| 0.747424 | 0.743303 |
| 0.711458 | 0.636445 |
| 0.817705 | 0.731674 |
| 0.777857 | 0.792857 |
| 0.672012 | 0.754902 |
| 0.677322 | 0.747424 |
| 0.743303 | 0.672681 |
| 0.781025 | 0.711458 |

$$\tilde{\tilde{x}}_D = \frac{1}{m}\left(\sum_{i=1}^{m} \tilde{x}_{i_D}\right) = 0.7309956; \quad \tilde{\tilde{y}}_M = \frac{1}{n}\left(\sum_{j=1}^{n} \tilde{y}_{jM}\right) = 0.718989$$

where $\tilde{\tilde{x}}_D$ and $\tilde{\tilde{y}}_M$ are the extremes of the given interval data.

$$S^2_{\tilde{x}_D} = \left(\frac{1}{m-1}\right)\left(\sum_{i=1}^{m}\left(\tilde{x}_{i_D} - \tilde{\tilde{x}}_D\right)^2\right) = 0.00063$$

$$S^2_{\tilde{y}_M} = \left(\frac{1}{n-1}\right)\left(\sum_{j=1}^{n}\left(\tilde{y}_{j_M} - \tilde{\tilde{y}}_M\right)^2\right) = 0.00029$$

$$t = \frac{\tilde{\bar{x}}_D - \tilde{\bar{y}}_M}{\sqrt{\frac{S^2_{\tilde{x}_D}}{m} + \frac{S^2_{\tilde{y}_M}}{n}}} = 1.253859$$

The tabulated value of $t$ at 5% leve of significance with 18 degrees of freedom is $t_\alpha = 1.73$

## 6 Conclusion

The Calculated value of $t$ is $|t| = 1.2539$ and the tabulated value of $t$ is $t_\alpha = 1.73$ Here, $|t| < t_\alpha \Rightarrow$ the null hypothesis $\tilde{H}_o$ is accepted and it can be conclude that the intensity of the infection of that two disease is same in that particular area. Although better decision can be arrived by using the method of ranking function obtained from the pivotal spot of fuzzy numbers, it is not an ultimate solution to all problems. Moreover, it has opened the door for the test of hypotheses involving TrFNs through their defuzzified forms such as rank etc. and ofcourse it needs further refinement and research to arrive a finer result.

## References


[1] L. A. Zadeh, “Fuzzy sets”, Information and Control, Vol. 8, pp. 338-353,1965.

[2] S. Abbasbandy and T. Hajjari, “Weighted trapezoidal approximation-preserving cores of a fuzzy number”, Computational and Mathematics with Applications, Vol. 59, pp. 3066-3077, 2010.

[3] S. Abbasbandy and B. Asady, “The nearest trapezoidal fuzzy number to a fuzzy quantity”, Application of Mathematics and Computation, Vol.156, pp. 381-386, 2004.

[4] Abhinav Bansal, “Trapezoidal Fuzzy Numbers (a, b, c, d): Arithmetic Behavior”, International Journal of Physical and Mathematical Sciences, pp. 39-44, 2011.

[5] B. F. Arnold, “Testing fuzzy hypotheses with crisp data”, Fuzzy Sets and Systems, Vol. 94, pp. 323-333, 1998.

[6] J. Chachi, S. M. Taheri and R. Viertl, “Testing statistical hypotheses based on fuzzy confidence intervals”, Forschungsbericht SM-2012-2, Technische Universitat Wien, Austria, 2012.

[7] P. Gajivaradhan and S. Parthiban, "Two sample statistical hypothesis test for trapezoidal fuzzy interval data", International Journal of Applied Mathematics and Statistical Sciences, Vol. 4 pp. 1124, 2015.
[8] S. Parthiban and P. Gajivaradhan, "Statistical Hypothesis on Industrial Applications through Ranks from COG of TrFNs", International Journal of Recent Technology and Engineering (IJRTE), Vol. 6, pp. 1116-1118, 2019.
[9] S. Parthiban and P. Gajivaradhan, "Statistical hypothesis test in three factor ANOVA model under fuzzy environments using trapezoidal fuzzy numbers", Bulletin of Mathematical Sciences and Applications, Vol.14, pp. 23-42, 2016.
[10]P. Grzegorzewski, "Testing statistical hypotheses with vague data", Fuzzy Sets and Systems, Vol. 112, pp. 501-510,2000.
[11]P. Grzgorzewski and E. Mrowka, "Trapezoidal approximation of fuzzy numbers", Fuzzy Sets and Systems, Vol. 153, pp. 115-135,2005.
[12] Iuliana Carmen "Statistical Hypothesis Testing Using Fuzzy Linguistic Variables", Fiabilitatesi Durabilitate-Fiability & Durability, Supplement, 1 (2012) Editura "Academica Brâncusi", Târgu Jiu, ISSN 1844-640X.
[13]T. S. Liou and M. J. Wang, "Ranking Fuzzy Numbers with Integral Value", Fuzzy Sets and Systems, Vol. 50, pp. 247-225,1992.
[14]Salim Rezvani, "Ranking Generalized Trapezoidal Fuzzy Numbers with Euclidean Distance by the Incentre of Centroids", Mathematica Aeterna, Vol.3, no. 2, pp. 103-114.
[15]Y. L. P. Thorani, P. Phani Bushan Rao and N. Ravi Shankar, "Ordering Generalized Trapezoidal Fuzzy Numbers", International Journal Contemporary Mathematical Sciences, Vol. 7, no. 12, pp. 555-573, 2012.
[16]R. Viertl, "Univariate statistical analysis with fuzzy data", Computational Statistics and Data Analysis, Vol. 51, pp. 33-147, 2006.
[17]Y. M. Wang, J.B. Yang, D. L. Xu and K. S. Chin, "On the centroids of fuzzy numbers", Fuzzy Sets and Systems, Vol. 157, pp. 919-926, 2006.
[18]H. C. Wu, "Analysis of variance for fuzzy data", International Journal of Systems Science, Vol. 38, pp. 235-246, 2007.
[19]H. C. Wu, "Statistical confidence intervals for fuzzy data", Expert Systems with Applications, Vol. 36, pp. 2670-2676, 2009.
[20]H. C. Wu," Statistical hypotheses testing for fuzzy data", Information Sciences, Vol. 175, pp. 30-56, 2005.

## Biographies

**K. S. Keerthika,** Research Scholar, Division of Mathematics, Department of Science and Humanities, Vignan's Foundation for Science, Technology and Research, Guntur, Andhra Pradesh, India, widely interested in field of Fuzzy Logic Applications in Probability and Statistics.

**S. Parthiban,** Associate Professor, Division of Mathematics, Department of Science and Humanities, Vignan's Foundation for Science, Technology and Research, Guntur, Andhra Pradesh, India. He is having more than 12 years of teaching and research experience in Pure and Applied Mathematics. And major area of research interest is Applications of Fuzzy Logic to Probability and Statistics and published many research papers in the above area.